

Aggregate demand

Determine the aggregate demand for good 1 in the following cases:

1. There are 15 identical consumers each with $m = 100$ and preferences represented by $u(x_1, x_2) = x_1 x_2$.
2. There are two consumers, A and B, with incomes $m_A = m_B = 50$ and preferences represented by $u^A = x_1 x_2$ and $u^B = \ln(x_1) + x_2$. Assume $p_2 = 5$.
3. There are two consumers, A and B, with incomes $m_A = 100$, $m_B = 50$ and preferences represented by $u^A = x_1 + x_2$ and $u^B = \min\{x_1, x_2\}$. Assume $p_2 = 3$.

Solution

1. The Marshallian demands for each individual are as follows:

$$x_1^m = \frac{100}{p_1} \frac{1}{2} = \frac{50}{p_1}$$

Summing up the Marshallian demands of the 15 individuals we get:

$$Q^D = \frac{50}{p_1} * 15 = \frac{750}{p_1}$$

2. The Marshallian demand for individual A is:

$$x_1^m = \frac{50}{p_1} \frac{1}{2} = \frac{25}{p_1}$$

Meanwhile, the Marshallian demand for individual B is:

$$x_1^m = \begin{cases} \frac{p_2}{p_1} & \text{if } m \geq p_2 \\ \frac{m}{p_1} & \text{if } m < p_2 \end{cases}$$

In this case, $m > p_2$ hence the Marshallian demand is:

$$\frac{p_2}{p_1} = \frac{5}{p_1}$$

Therefore, the aggregate demand is:

$$Q^D = \frac{25}{p_1} + \frac{5}{p_1} = \frac{30}{p_1}$$

3. The Marshallian demands for individual A are:

$$x_1^m = \begin{cases} \frac{m_A}{p_1} & \text{if } p_1 < p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases}$$

While for $p_1 = p_2$, the demand can take any value $\in [0, \frac{m_A}{p_1}]$. For individual B, the Marshallian demand is:

$$x_1^m = \frac{m_B}{p_1 + p_2}$$

Thus, the aggregate demand function is divided depending on whether p_1 is greater or smaller than p_2 :

$$Q^D = \begin{cases} \frac{m_A}{p_1} + \frac{m_B}{p_1 + p_2} & \text{if } p_1 < p_2 \\ 0 + \frac{m_B}{p_1 + p_2} & \text{if } p_1 > p_2 \end{cases}$$

Lastly, in the case that $p_1 = p_2$:

$$Q^D = x_1^m + \frac{50}{6}$$

Where $x_1^m \in [0, \frac{100}{3}]$.